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Technical Note

ON THE FOUNDATIONS OF DYNAMICAL ANALYSES
OF BOOST VEHICLES, II: THE INHOMOGENEOUS PROBLEM

by

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PREFACE

This Technical Note was prepared for the National Aeronautics and Space Administration under Contract NASr-21(03), which is monitored by The Dynamic Loads Division, Langley Research Center. The results reported here are an extension and elaboration of those presented in AR-110-NASA, RAND's final report on the contract, which was submitted to Dr. Thomas L. K. Smull, Director of Grants and Research Contracts, NASA, on 26 December 1963. The concern here is with the objectives of Phase II of this research program, namely, the characterization of the dynamics of boost vehicles in the presence of external excitations. The primary investigations concerned the question of whether linearized analyses of boost vehicles provide acceptable predictive techniques for determining the dynamical environment of the vehicle when the governing differential equations are nonlinear and driven by external (stochastic) excitations.

1. The Governing Equations in Perturbation Form

The mathematical basis for the present considerations is the complete nonlinear perturbation equations of a boost vehicle as developed in Reference [1]. We give here a brief review of their derivation together with certain additional results which may be useful in conjunction with the homogeneous problem.

Let $\{X(t)\}$ denote the column matrix of dynamical variables that appears in the trajectory (rigid body) equations of a boost vehicle when these equations are reduced to first-order differential equations, and let $\{Y(t)\}$ denote the remaining dynamical variables (again referred to first-order differential representations). The dynamical state of a boost vehicle is thus described by the system

$$(1.1) \quad \frac{d}{dt} \begin{Bmatrix} \{X(t)\} \\ \{Y(t)\} \end{Bmatrix} = \begin{Bmatrix} \{F(X(t), Y(t), t)\} \\ \{G(X(t), Y(t), t)\} \end{Bmatrix}.$$

Since the trajectory equations usually ignore certain terms, it is assumed that the trajectory equations are given by

$$(1.2) \quad \frac{d}{dt} \{\tilde{X}(t)\} = \{H(\tilde{X}(t), t)\},$$

where

$$(1.3) \quad \{F(\tilde{X}(t), 0, t)\} = \{H(\tilde{X}(t), t)\} + \{K(\tilde{X}(t), t)\}.$$

Here $\{\tilde{X}(t)\}$ denotes a solution of the trajectory equations (1.2), and the column matrix $\{K(\tilde{X}(t), t)\}$ represents the terms that are ignored in the trajectory calculations.

Since trajectory calculations have been highly refined over the past decade, it is assumed that an exact solution, $\{\tilde{X}(t)\}$, of the trajectory equations (1.2) is known. As shown in Ref. [1], an expansion of the dynamical state about the trajectory state, that is

$$\{X(t)\} = \{\tilde{X}(t)\} + \{x(t)\}, \quad \{Y(t)\} = \{0\} + \{y(t)\},$$

leads to the following system of equations:

$$(1.4) \quad \frac{d}{dt} \begin{Bmatrix} \{x(t)\} \\ \{y(t)\} \end{Bmatrix} = \begin{Bmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{Bmatrix} \begin{Bmatrix} \{x(t)\} \\ \{y(t)\} \end{Bmatrix} + \begin{Bmatrix} \{K(\tilde{X}, t)\} \\ \{G(\tilde{X}, 0, t)\} \end{Bmatrix} + \star.$$

In this system, the matrices \underline{A} , \underline{B} , \underline{C} , \underline{D} are known functions of the rigid-body trajectory solution $\tilde{X}(t)$ (see Eq. (34) of [1]), and hence are functions of the time, and the second- and higher-order terms in the variables $\{x\}$, $\{y\}$ are denoted by \star .

Noting that $\{K(\tilde{X}, t)\}$, rather than $\{F(\tilde{X}, 0, t)\}$, appears in these equations, the nonlinearities involved in the trajectory equations have been eliminated. This is indeed a marked simplification since the trajectory equations are the principal sources of kinematic nonlinearity.

To simplify the ensuing discussion, it is useful to introduce the following notation. Set

$$(1.5) \quad \{u(t)\} = \begin{Bmatrix} \{x(t)\} \\ \{y(t)\} \end{Bmatrix},$$

$$(1.6) \quad \{g(t)\} = \begin{Bmatrix} \{K(\bar{X}, t)\} \\ \{G(\bar{X}, 0, t)\} \end{Bmatrix},$$

and

$$(1.7) \quad \underline{W} = \begin{pmatrix} \underline{A} & \underline{B} \\ \underline{C} & \underline{D} \end{pmatrix}.$$

Thus, if the second- and higher-order terms (the \bar{X} -terms in (1.4)) are denoted by $\{f(u, t)\}$, the system (1.4) becomes

$$(1.8) \quad \frac{d}{dt} \{u(t)\} = \underline{W}(t) \{u(t)\} + \{g(t)\} + \{f(u, t)\},$$

with

$$(1.9) \quad \{f(0, t)\} = \{0\}.$$

The work reported in Ref. [1] dealt with the homogeneous problem, namely, the system

$$(1.10) \quad \frac{d}{dt} \{u(t)\} = \underline{W}(t) \{u(t)\} + \{f(u, t)\}$$

that is obtained from the system (1.8) by deleting the inhomogeneous terms $\{g(t)\}$. (The condition expressed by (1.9) shows that (1.10) is the homogeneous system corresponding to (1.8).) Physically, this problem deals with the stability and load environment of a boost vehicle in the absence of any external environmental excitations (wind, shears, gusts, etc.) that are not included in the trajectory equations (1.2). Since the analysis of boost vehicles, at least in the preliminary design phase, does not usually rely on the nonlinear system (1.10), but rather on the linearized version

$$(1.11) \quad \frac{d}{dt} \{h(t)\} = \underline{w}(t) \{h(t)\},$$

and even more often on the linear version with constant coefficients,

$$(1.12) \quad \frac{d}{dt} \{\bar{h}(t)\} = \underline{w}(T = \text{constant}) \{\bar{h}(t)\},$$

two basic questions must be answered. The first is: Under what conditions does the linear system (1.11) exhibit the same behavior as the nonlinear system (1.10)? The second is: Under what conditions does the linear system with constant coefficients (1.12) exhibit the same behavior as the nonlinear system (1.10)?

These questions are answered in Ref. [1]. Definite

criteria are given, which ensure that the linearized analyses adequately reflect the behavior of the nonlinear system. Explicit formulae are also presented by which the differences between the solutions of systems (1.11) and (1.12) may be computed during the initial phase of the boost process. The reason for this inclusion is that the basic results on the equivalence between systems (1.10), (1.11), and (1.12) are asymptotic--valid for large times.

A particularly useful extension of the results of Ref. [1] is as follows. Let S be the point set in the N -dimensional space with radius vector $\{u(t)\}$ that is the initial-condition set for the complete homogeneous perturbation equations (1.10). By the results given in Sec. 4 of Ref. [1], we know that

$$(1.13) \quad \lim_{t \rightarrow T} \|\{u(t) - \{h(t)\}\| = 0$$

provided

$$(1.14) \quad \{h(0)\} = \{u(0)\} + \{K(T)\} ,$$

$$(1.15) \quad \{K(T)\} = \int_0^T \underline{H}^{-1}(\lambda) \{f(u(\lambda), \lambda)\} d\lambda .$$

Here $\underline{H}(t)^{-1}$ denotes the inverse of the fundamental normal matrix $H(t)$ associated with the linear system (1.11); that is,

$$(1.16) \quad \frac{d}{dt} \underline{H}(t) = \underline{W}(t) \underline{H}(t), \quad \underline{H}(0) = \underline{I},$$

and

$$(1.17) \quad \{h(t)\} = \underline{H}(t) \{h(0)\}.$$

This result tells us that we can find an initial condition (1.14) for the linear system (1.11) such that the linear system and the nonlinear system (1.10) have the same dynamical state at time T . Unfortunately, the column matrix $\{K(T)\}$ which determines the appropriate initial condition for the linear system depends on the unknown solution $\{u(t)\}$ of the nonlinear system (1.10). However, Eq. (67) of Ref. [1] gives

$$(1.18) \quad \|\{K(T)\}\| \leq N^{1/2} \int_0^T \|\underline{H}^{-1}(\lambda)\| \cdot |g(\lambda)| \cdot \|\{u(\lambda)\}\| d\lambda,$$

where $g(\lambda)$ is the Lipschitz function for the nonlinearity $\{f(u(t), t)\}$:

$$|f_1(u_1, \dots, u_N, t) - f_1(v_1, \dots, v_N, t)| \leq g(t) \sum_{j=1}^N |u_j - v_j|.$$

In terms of the bounds for $\|\underline{H}^{-1}(\lambda)\|$ and $\|\{u(\lambda)\}\|$ given by Eqs. (55) and (56) (which depend only on the known functions $g(t)$ and $\underline{W}(t)$, the right-hand side of (1.18) can be replaced by an upper bound which is a known functional of $g(t)$ and $\underline{W}(t)$. This gives, say,

$$(1.19) \quad \| \{K(T)\} \| \leq B(T) ,$$

and $B(T)$ is a monotone nondecreasing function of T with $B(0) = 0$. If, for each point P of S , we construct an N -dimensional sphere $R_p(T)$ with center at P and with radius $B(T)$, then the initial point Q given by (1.14) for the linear system for which (1.13) holds will be such that $Q \subset R_p(T)$. Hence, for the point set (see Fig. 1)

$$(1.20) \quad H(T) = \bigcup_{P \in S} R_p(T)$$

as the initial-condition point set for the linear system (1.11), the solution manifold of the linear system will contain solutions which satisfy (1.13) at time T . From this and the monotone property of $B(T)$, it follows that if the design of the boost vehicle is acceptable for all solutions of the linear system (1.11) that result from the initial condition set (1.20) for given T , then the design is acceptable according to the actual nonlinear system (1.10) for all values of the time less than or equal to T . In addition, if we substitute (1.14) into (1.17), we have

$$(1.21) \quad \{h(t)\} = \underline{H}(t) \{u(0)\} + \underline{H}(t) \{K(T)\} ,$$

where $\underline{H}(t) \{u(0)\}$ is what would be obtained from the linear system if the initial conditions for the complete

nonlinear system were used. It thus follows that

$$\| \{h(t)\} - \underline{H}(t)\{u(0)\} \| \leq \| \underline{H}(t) \| \cdot \| \{K(T)\} \| ,$$

and hence, by Eq. (55) of Ref. [1] and (1.19),

$$(1.22) \quad \| \{h(t)\} - \underline{H}(t)\{u(0)\} \| \leq B(T) \exp \left\{ N^{1/2} \int_0^t \| \underline{W}(\tau) \| d\tau \right\} .$$

This last result shows the effect of the different initial conditions that must be used for the linear system; that is, the effect of using the set $H(T)$ rather than S .

2. The Inhomogeneous Problem

The natural counterpart of the studies reported in Ref. [1] is an examination of the mathematical and physical foundations of the dynamical analysis of boost vehicles when the dynamical state of the vehicle is a consequence of external excitations--the inhomogeneous problem. Of primary concern are the (random) excitations of the system that result from the side wind, gust, and wind shear environment to which the vehicle is subjected during its ascent. During the course of this study, a bibliography of the aerodynamic environments of boost vehicles was prepared. This is given in Appendix I.

3. The Rigid-Body Excitation Problem

The governing system of equations appertaining to the inhomogeneous problem is that given by Eq. (1.8).

Since the system (1.8) results from the complete dynamical system (1.1) when an expansion is made about a known trajectory solution $\{\tilde{X}(t)\}$ of the system (1.2), the first problem to be resolved is to determine just what excitation is assumed for the rigid-body (trajectory) system. If the excitation of the complete system (1.1) is described explicitly, the answer is obvious; the excitation to be used in the rigid-body system (1.2) is the explicit rigid-body excitation. On the other hand, if the excitation is the result of a random process, the answer is by no means obvious. This is due to the fact that the rigid-body solution $\{\tilde{X}(t)\}$ is assumed to be an explicit known function of the time, and hence, since $\{\tilde{X}(t)\}$ is a solution of (1.2), the excitation part of the matrix $\{H(\tilde{X}(t), t)\}$ must be an explicit function of the time and $\{\tilde{X}(t)\}$. Since the trajectory equations (1.2) are not assumed to contain all terms that would result from the complete system (1.1) on setting $\{Y(t)\}$ equal to zero, there is no problem of consistency involved in taking $\{H(\tilde{X}(t), t)\}$ as an explicit function of the time and $\{\tilde{X}(t)\}$, even though $\{F(\tilde{X}(t), 0, t)\}$ results in part from a random process. The problem of determining the appropriate external excitation part of $\{H(\tilde{X}(t), t)\}$ when the system is subjected to an external

excitation that results from a random process is called the rigid-body excitation problem.

There are two principal mitigating circumstances in the resolution of the rigid-body excitation problem. The first is that it is desirable to include as much of the excitation in the rigid-body equations as possible. The resulting trajectory solution would thereby contain the principal effects of the excitation, and hence one might hope that the solution of the system (1.8) would predict only small differences between the true dynamical state and the dynamical state represented by $\{\tilde{X}(t)\}$. The second is that, from the statistical point of view, it is desirable that the rigid-body excitation function be taken as the mean of the actual random excitation function. If this is done, Eq. (1.3) shows that the excitation matrix $[K(\tilde{X}(t), t)]$ will contain the random portion of the excitation and in addition, this random portion will have zero mean.

A solution of the rigid-body excitation problem that holds good in all instances is obviously impossible. A solution will have to be made in each particular design situation, and will depend on the combination of many other circumstances in addition to the two cited above. Whatever the particular solution of this problem, it is most important to note that $[K(\tilde{X}(t), t)]$, not $[H(\tilde{X}(t), t)]$, contains any and all statistically specified excitations.

4. The Nonlinearity Problem

The governing system of equations is

$$(4.1) \quad \frac{d}{dt} [u(t)] = \underline{W}(t) [u(t)] + [f(u, t)] + [g(t)] ,$$

where

$$(4.2) \quad [f(0, t)] = [0] , \left([f(u, t)], [u(t)] \right) \Big|_{u=0} = 0 ,$$

and where the excitation matrix $[g(t)]$ is given by Eq. (1.6):

$$(4.3) \quad [g(t)] = \begin{Bmatrix} [K(\tilde{X}, t)] \\ [G(\tilde{X}, 0, t)] \end{Bmatrix} .$$

Under linearization, the system becomes

$$(4.4) \quad \frac{d}{dt} [\bar{u}(t)] = \underline{W}(t) [\bar{u}(t)] + [g(t)] .$$

The same basic question arises in conjunction with the inhomogeneous systems (4.1) and (4.2) as arose with the homogeneous systems: Under what conditions will the dynamical state predicted by the linear system (4.4) provide an acceptable description of the dynamical state that would be predicted by the actual nonlinear system (4.1)? The following alternative question was also considered. Under what conditions can we find a matrix $\underline{V}(t)$ and a column

matrix $\{k(t)\}$ such that a solution $\{s(t)\}$ of the linear system

$$(4.5) \quad \frac{d}{dt} \{s(t)\} = \underline{v}(t) \{s(t)\} + \{k(t)\}$$

provides an acceptable description of the dynamical state of the non-linear system satisfying the governing system of equations (4.1)?

Although a substantial portion of the effort during the study was expended on attempts to resolve the above questions, no analytic methods could be found whereby even partial answers could be obtained. The results of all attempts were invariably negative. The most we can attempt to do here is to give some indication as to the sources of the difficulty.

First, the linear system (4.4) contains only the excitation function as determined by the trajectory solution $\{\tilde{X}(t)\}$. This is seen from Eqs. (4.3). On the other hand, even though $\{f(0, t)\} = \{0\}$, $\{f(u, t)\}$ contains terms that depend on the external excitation. Thus, the nonlinear system (4.1) is intrinsically different from the linear system (4.4) since the nonlinear terms in (4.1) contribute to the excitation in addition to the contributions from $\{g(t)\}$. Stated another way, the linear system (4.4) has its excitation uniquely determined by the external environment and the corresponding rigid-body trajectory--a situation

obviously contradicted by the fact that the external excitation of the actual physical system is determined by the complete dynamical state, not just that part of the dynamical state described by the rigid-body trajectory solution.

Second, even assuming that the system (4.1) is deterministic (the system is subjected to explicit, known excitations, not random processes), no general conditions are known in the present literature nor could any be found under which the linear system (4.4) or (4.5) provides adequate information concerning the dynamical state of the nonlinear system (4.1). On the other hand, numerous examples can be found in which the linear system (4.4) or (4.5) converges while the nonlinear system diverges.

Finally, if the nonlinear system (4.1) is driven by a random process, the situation is only compounded. The problem of determining the mean and variance of the output of a nonlinear system driven by a random process, by comparison with the mean and variance associated with the same system linearized about a motion of the homogeneous nonlinear system is beyond the state of the art at present.

As an example, consider the problem of a one-dimensional Markov continuous-parameter process $u(t)$ such that the increment between times t_2 and t_1 is a sum of small increments $du(t)$, each which is Gaussian with mean $m dt$ and variance $\sigma^2 dt$ (unless otherwise stated, we shall use

the notation of Ref. 2, which see). In particular, we consider

$$(4.6) \quad du(t) = m[t, u(t)]dt + \sigma(t)dy(t),$$

where the $y(t)$ process is the Brownian-motion process with variance parameter 1; that is, it is a real Gaussian process with independent increments and

$$E[y(t_2) - y(t_1)] = 0, \quad E[[y(t_2) - y(t_1)]^2] = |t_2 - t_1|.$$

If $p(s, \xi; t, \eta)$ denotes the transition probability-distribution function from the state $t = s, u = \xi$ to the state $t = T, u = \eta$, then we must solve the Kolmogonov-Fokker-Plank diffusion equations:

$$(4.7) \quad \left\{ \begin{array}{l} \frac{\partial p'(s, \xi; T, \eta)}{\partial T} + \frac{\partial}{\partial \eta} [m(T, \eta)p'(s, \xi; T, \eta)] \\ \quad - \frac{\sigma(T)^2}{2} \frac{\partial^2 p'(s, \xi; T, \eta)}{\partial \eta^2} = 0, \\ p' = \frac{\partial p(s, \xi; T, \eta)}{\partial \eta}, \\ p(s, \xi; T, \eta) \Big|_{T=s} = \begin{cases} 1 & \eta > \xi, \\ 0 & \eta < \xi. \end{cases} \end{array} \right.$$

If we set

$$(4.8) \quad m(t, u(t)) = Au(t) + f(t, u(t)),$$

the linearized equation corresponding to (4.6) is

$$(4.9) \quad dv(t) = A(t)v(t) + \sigma(t) dy(t) .$$

Hence, a comparison of the linear equation (4.9) with the nonlinear equation (4.6) involves a comparison of the solution of $p(s, \xi; T, \eta)$ of (4.7) with the solution $P(s, \xi; T, \eta)$ to the equations that are obtained from (4.7) when $m(T, \eta)$ is replaced by $A(T)y$, namely

$$(4.10) \quad \begin{cases} \frac{\partial P'}{\partial T} + \frac{\partial}{\partial \eta} [A\eta p'] - \frac{\sigma^2}{2} \frac{\partial^2 P'}{\partial \eta^2} = 0 , \\ P' = \partial P / \partial \eta , \\ P(s, \xi; T, \eta) \Big|_{T=s} = \begin{cases} 1 & \eta > \xi , \\ 0 & \eta < \xi . \end{cases} \end{cases}$$

This in itself is a very difficult problem, not to mention the fact that for even the simplest nonlinearities the system (4.7) is analytically untractable.

The converse problem, namely that of determining the best linear system, may be stated as follows: Find functions $A(t)$ and $\sigma^*(t)$ such that

$$\int_s^\infty [p(s, \xi; T, \lambda) - p^*(s, \xi; T, \lambda)]^2 d\lambda$$

is minimal, where p^* is a solution of (4.10) with $\sigma(t)$ replaced by $\sigma^*(t)$.

5. The Representation Problem

The remainder of the effort centered around the problem of how to represent the random wind profiles to which a boost vehicle is subjected. This problem is intimately connected with the nonlinearity problem--the appropriate representation process being determined by the manner of analysis and the representation of the system to which the random process is applied as an excitation.

In view of the known variations of the wind profiles with altitude, it is evident that the wind profiles cannot be represented as a stationary process (as a function of altitude), although the winds may possibly be considered as stationary processes if the altitude is held fixed. The next simplest process is one that is weakly stationary (quasi-stationary, stationary in the wide sense); that is, although the probability-distribution function of the winds is not invariant under translation with respect to altitude h , the autocorrelation function $R(h_1, h_2)$ is such that $R(h_1, h_2) = \underline{R}(h_1 - h_2)$ (see [2], Chap. II, Art. 8).

If we define the function $f(\lambda)$ by

$$(5.1) \quad \underline{R}(h_1 - h_2) = \int_{-\infty}^{\infty} e^{i\lambda(h_1 - h_2)} f(\lambda) d\lambda,$$

then the random horizontal wind profile as a function of h assumes the form

$$(5.2) \quad v(h) = \int_{-\infty}^{\infty} e^{ih\lambda} \sqrt{f(\lambda)} dy(\lambda) ,$$

where, as before, the $y(t)$ process is the Brownian-motion process with variance parameter 1. Thus, if $h_1(t)$ and $h_2(t)$ are the altitudes of the nose and tail of the vehicle at time t , as obtained from the rigid-body equations, the excitation function of the vehicle due to horizontal winds may be represented as

$$(5.3) \quad [g(t)] = \int_{h_1(t)}^{h_2(t)} [Q(h, t)] V(h) dh.$$

Here $Q_i(h, t)$ denotes the load on the i -th dynamical variable per unit of horizontal wind velocity per unit of vehicle length when the length along the vehicle is written in terms of a difference between h and $h_1(t)$. Introducing the function

$$\phi(h, t) = \begin{cases} 1 & \text{if } h_1(t) \leq h \leq h_2(t), \\ 0 & \text{otherwise,} \end{cases}$$

Eq. (5.3) can be written as

$$(5.4) \quad [g(t)] = \int_{-\infty}^{\infty} [Q(h, t)] V(h) \phi(h, t) dh.$$

When Eq. (5.2) is substituted into (5.4), we then have

$$(5.5) \quad [g(t)] = \int_{-\infty}^{\infty} [T(\lambda, t)] \sqrt{f(\lambda)} d\lambda ,$$

where

$$(5.6) \quad [T(\lambda, t)] = \int_{-\infty}^{\infty} [q(h, t)] \phi(h, t) e^{i h \lambda} dh.$$

Hence, in the case of a weakly stationary process, the excitation matrix $[g(t)]$ assumes the particularly simple form (5.5), since $[T(\lambda, t)]$ is the matrix of Fourier transforms of known functions, as shown by (5.6). Even in this simple case, however, it is evident from (5.5) that the process $[g(t)]$ is no longer weakly stationary. This will be the rule rather than the exception, since the rapid acceleration, attitude, and altitude changes of boost vehicles will always result in the vehicle sensing a significantly different process than that which may provide an acceptable representation process for the aerodynamic environment as a function of altitude.

Even if stationarity, weak stationarity, or some equivalent assumption is made, it turns out that little actual progress can be made on the general problem of the dynamics of boost vehicles. The effects of the nonlinearities and the obvious deficiencies in the linearization are such that no general procedure appears acceptable in all cases. At the present state of the art, each particular design must be evaluated in its own right. The only present procedure by which confidence can be established in the design of boost vehicles under wind excitation is that of

detailed digital simulation with the best available sample functions for the wind excitations. In fact, the unresolved nature of the nonlinear problem shows that at present no one method of representation of the environment is preferable to any other, since neither method could lead to a design validation without detailed digital simulation.

6. The Load Environment

Suppose that it were possible to obtain a solution $\{u(t)\}$ of the nonlinear problem (4.1) under a stochastic excitation $\{g(t)\}$. By adding $\{u(t)\}$ and the corresponding trajectory state, we would then know the complete dynamical state, say $\{U(t)\}$, of the boost vehicle. Correspondingly, the dynamic stress state (bonding moments, etc.) $L(x, t)$ at a point on the vehicle a distance x from the nose of the vehicle would be given by

$$(6.1) \quad L(x, t) = [M(x, t)]^T \{U(t)\},$$

where T denotes transpose and $M_i(x, t)$ denotes the load contribution at x due to the i -th dynamical variable. If $c(x, t)$ denotes the allowable load environment, the vehicle will not fail so long as

$$(6.2) \quad |L(x, t)| < c(x, t).$$

Ideally, we should like to calculate the probability of (6.2) occurring for all x in the vehicle and for all t in the time interval of interest. Unfortunately, this problem is also analytically untractable; the most that can be done is to attempt to apply Monte Carlo simulation through detailed digital calculations of solutions of (4.1) for a representative family of wind profiles. The situation

is analogous to the problem of calculating the probability distribution of the number of maxima per unit of time of a random function of one variable. The result is unknown, even supposing a Gaussian stationary process, yet the mean number of maxima which exceed any given threshold is given by the well-known formula of Rice [3]. It thus turns out that even if we could obtain acceptable solutions representing the dynamical state of a boost vehicle when subjected to stochastic representation, the ability to calculate the probability of design would be lacking.

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Appendix I

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